MODELLING OF ROTATING DISK VIBRATION WITH FIXED BLADES

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Abstract: The paper presents the method for the mathematical modelling of bladed disk vibrations. The blades are considered as one dimensional continuum fixed with three-dimensional elastic disk centrally clamped into turbine rotor rotating with constant speed. A dynamic analysis and optimization of the bladed disk vibration with damping elements requires a development of the efficient method pointing to the bladed disk model with relatively small DOF number. In the future, such model will enable simulations of the whole system nonlinear vibration influenced by slip contact interactions in inner couplings between blades.

1. INTRODUCTION

The aim of this article is to develop suitable methodology for vibration modelling of the rotating flexible disk with fixed blades (Fig. 1). The method is based on the discretization of 3D rotating disks [1] and 1D blades [2] by FEM. The rigid couplings between disk and blade feet are considered. The presented contribution is the first preliminary step for the research of dynamic behaviour of the bladed disks with damping caused by slip contact interactions in inner couplings between blade shroud.

Fig. 1 Scheme of the rotating bladed disk
Analytical and experimental research performed in Institute of Thermomechanics AS CR [3] on two test parallel beams connected by a damping element turns out that equivalent linearized coefficients of damping can be used in the first approximation to describe friction behaviour. Nevertheless a large DOF number of bladed disk mathematical model requires reduction. Therefore we will apply a modal synthesis method with condensation [4] to the modelling of bladed disks in the meantime without inner couplings between blades.

2. MATHEMATICAL MODELLING OF THE DISK WITH FIXED BLADS

The rotating bladed disk (Fig. 1) can be generally decomposed into a disk (subsystem $D$) and separated blades (subsystems $B_i$, $i = 1, 2, \ldots, r$). We assume that the disk is centrally clamped into rotor rotating with constant angular velocity $\omega$ around its $y$–axis. According to the derivation presented in [5] the disk can be discretized in the rotating $x$ $y$ $z$ – coordinate system using linear isoparametric hexahedral finite elements. The equation of motion can be written in a configuration space defined by the vector

$$ q_D = [\ldots, u_j^{(F)}, v_j^{(F)}, w_j^{(F)}, \ldots, u_j^{(C)}, v_j^{(C)}, w_j^{(C)}, \ldots]^T \in R^{n_D} $$

of nodal $j$ displacements (see Fig. 1) in direction of rotating axis $x$, $y$, $z$. The disk nodes are classified into free nodes (superscript $F$) and coupled nodes (superscript $C$) on the outer and inner surface of the blade foots. The mathematical model of the disk was derived in [1] using Lagrange’s equations in the form

$$ M_D \ddot{q}_D(t) + \omega G_D \dot{q}_D(t) + \left( K_D - \omega^2 K_{dd} \right) q_D(t) = \omega^2 f_D, $$

where $M_D$, $K_D$ and $K_{dd}$ are symmetric mass, static stiffness and dynamic softening matrices and skew-symmetric matrix $\omega G_D$ expresses gyroscopic effects.

The single blades are modelled as one dimensional continuum linked with rigid shroud body in its centre of gravity. The mathematical model of the uncoupled blade $i$ with shroud in configuration space of its blade node displacements in the direction of rotating axis $x_i$, $y_i$, $z_i$ and of small angular displacements of the blade cross sections

$$ q_{B,i} = [\ldots, u_j, v_j, w_j, \varphi_j, \theta_j, \psi_j, \ldots]^T \in R^{n_B}, i = 1, 2, \ldots, r, $$

has the form [2], [6]

$$ M_B \ddot{q}_{B,i}(t) + \omega G_B \dot{q}_{B,i}(t) + \left( K_B - \omega^2 K_{db} + \omega^2 K_{dd} \right) q_{B,i}(t) = \omega^2 f_B, \quad i = 1, 2, \ldots, r, $$

where blade matrices $M_B$, $K_B$, $K_{db}$ and $G_B$ have an identical meaning with matrices of the disk and matrix $\omega^2 K_{dd}$ expresses a centrifugal blade stiffening.

The vector of generalized coordinates of the disk can be partitioned according to (1) as

$$ q_D = \begin{bmatrix} q_D^{(F)} \\ q_D^{(C)} \end{bmatrix}, \quad q_D^{(F)} \in R^{n_D^{(F)}}, \quad q_D^{(C)} \in R^{n_D^{(C)}}. $$
The displacements of the coupled disk nodes on condition of rigid blade foots modelled as a disk part can be expressed by displacements of referential nodes $R_i$ which are identical with the first blade nodes $j=1$ at blade roots (see Fig. 1). This relation between coupled displacements corresponding to blade $i$ is

$$
\begin{bmatrix}
\theta_j^{(C)} \\
\nu_j^{(C)} \\
U_j^{(C)}
\end{bmatrix}_D
= 
\begin{bmatrix}
\cos \alpha_i & 0 & \sin \alpha_i \\
0 & 1 & 0 \\
-\sin \alpha_i & 0 & \cos \alpha_i
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 & -z_j & -y_j \\
0 & 1 & 0 & z_j & 0 \\
0 & 1 & 0 & y_j & -x_j
\end{bmatrix}
\begin{bmatrix}
\mu_i \\
v_i \\
w_i \\
\varphi_i \\
\dot{\varphi}_i
\end{bmatrix}.
$$

(6)

or shortly

$$
q_j^{(C)} = T_{a_i} T_j q_{i,j} \quad i = 1, 2, \ldots, r,
$$

(7)

where $x_j, y_j, z_j$ are coordinates of the coupled disk node $j$ on the rigid blade foots in coordinate system $x_i, y_i, z_i$ of the blade $i$ with the origin in the first blade node and $\alpha_i$ is the angle between the rotating disk axis $x$ and the rotating blade axis $x_i$.

The complete transformation between displacements of coupled nodes of the disk on the blade foots and the referential nodes $R_i$ of all blades can be expressed in the matrix form

$$
\begin{bmatrix}
\vdots \\
q_j^{(C)} \\
\vdots
\end{bmatrix}
= 
\begin{bmatrix}
\vdots \\
T_{a_i} T_j \\
\vdots
\end{bmatrix}
\begin{bmatrix}
q_{1,j} \\
q_{2,j} \\
\vdots
\end{bmatrix}
\Rightarrow q_j^{(C)} = T_{D,R} q_R .
$$

(8)

The global transformation rectangular matrix $T_{D,R} \in R^{n_D \times n_R}$ describes the linkage between the disk ($D$) and the blade rim ($R$). Coordinates of vector $q_R$ express displacements of the blade nodes $j = 1, 2, \ldots, N$ according to (3) in coordinate systems $x_i, y_i, z_i$ (see Fig. 1) in order of blades (for $i = 1, 2, \ldots, r$)

$$
q_R = [q_{r_1}^T, q_{r_2}^T, \ldots, q_{r_N}^T]^T \in R^{n_R}, \quad n_R = 6Nr ,
$$

(9)

where $r$ is the blade number.

The motion equations of the fictive undamped system assembled from uncoupled subsystems–disk with blade foots and isolated blades with shroud – in configuration space

$$
q = \left[ q_D^{(F)} \right]^T \left[ q_D^{(C)} \right]^T \left[ q_R \right]^T \in R^{n_D + n_R}
$$

(10)

can be formally rewritten as

$$
M \ddot{q}(t) + \omega G \dot{q}(t) + \left(K - \omega^2 K_d + \omega^2 K_m\right)q(t) = \omega^2 f .
$$

(11)

According to mathematical models (2) and (4), matrices have the block-diagonal form

$$
X = \text{diag} (X_D, X_B, X_B, \ldots, X_B), \quad X = M, K, K_d, K_m ,
$$

(12)

whereas $K_{m0} = 0$ and $f = [f_D^T, f_B^T, f_B^T, \ldots, f_B^T]^T$. 
3. CONDENSED MATHEMATICAL MODEL OF THE BLADED DISK

The number of free node elastic coordinates \( q^{(F)}_D \) of the disk is very large for future dynamic analysis of the bladed disk with dry friction elements. Hence, disk DOF number corresponding to free node coordinates and blades DOF number is desirable to reduce by use of the modal condensation [7].

Let modal properties of the conservative model of the non-rotating disk with blade foots isolated from blades in cross-sections passing through referential nodes \( R_i \) be characterized by spectral \( \Lambda_D \) and modal \( V_D \) matrices. These matrices satisfy the orthogonality and norm conditions
\[
V_D^T M_D V_D = E, \quad V_D^T K_D V_D = \Lambda_D,
\]
where \( E \) is unit matrix. The modal matrix of the disk can be rearranged into the block form
\[
V_D = \begin{bmatrix} m V^F_D & s V^F_D \\ m V^C_D & s V^C_D \end{bmatrix}
\]
corresponding to decomposition (5) and eigenvectors are separated into frequency lower eigenvectors (so called master – superscript \( m \)) and frequency higher eigenvectors (so called slave – superscript \( s \)). The vector \( q^{(F)}_D \), corresponding to free disk nodes, can be approximately transformed in the form
\[
q^{(F)}_D = m V^F_D x_D, \quad (15)
\]
where \( m V^F_D \in R^{m_D \times m_D} \) is the modal submatrix corresponding to free disk displacements and frequency lower eigenmodes. Higher natural modes usually contribute less to the disk deformation and their influence can be neglected.

The vectors \( q_B, \) corresponding to blade displacements, can be transformed by means of a modal submatrix of one isolated blade as
\[
q_{B,i} = m V_B x_i, \quad i = 1, 2, ..., r, \quad (16)
\]
where the modal submatrix \( m V_B \in R^{m_B \times m_B} \) of master mode shapes of the blade satisfies the orthogonality and norm conditions
\[
m V^T_B M_B m V_B = E, \quad m V^T_B K_B m V_B = m \Lambda_B. \quad (17)
\]

The vector \( q \) of the fictive system in consequence of the coupling (8) and modal transformations (15) and (16) can be transformed into new vector \([x_D^T x_R^T]^T\) of the dimension \( m = m_D + r m_B \). The transformation is given by
\[
\begin{bmatrix} q^{(F)}_D \\ q^{(C)}_D \\ q_B \end{bmatrix} = \begin{bmatrix} m V^{(F)}_D & 0 \\ 0 & T_{D,R} m V^R_R \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_D \\ x_R \end{bmatrix} \text{ or shortly } q = T x, \quad (18)
\]
where matrix \( m V_R = diag (m V_B) \in R^{m_B \times m_B} \) and \( m_R = r m_B \).
The condensed mathematical model of the bladed disk after elimination of the coupled displacements takes the form

$$\ddot{\mathbf{M}} \ddot{\mathbf{x}}(t) + \omega \dddot{\mathbf{G}} \dot{\mathbf{x}}(t) + \left( \mathbf{K} - \omega^2 \mathbf{K}_d + \omega^2 \mathbf{K}_w \right) \mathbf{x}(t) = \omega^2 \mathbf{f},$$  \hspace{1cm} (19)

where transformed matrices are

$$\tilde{\mathbf{X}} = \mathbf{T}^T \mathbf{X} \mathbf{T}, \quad \mathbf{X} = \mathbf{M}, \mathbf{G}, \mathbf{K}, \mathbf{K}_d, \mathbf{K}_w, \quad \tilde{\mathbf{f}} = \mathbf{Tf}.$$  

The condensed model (19) has much lower DOF number compared to noncondensed (full) model gained from the fictive model (11) by the transformation (18) with the modified transformation matrix

$$\mathbf{T}_f = \begin{bmatrix} \mathbf{E}_D & 0 \\ 0 & \mathbf{T}_{D,R} \\ 0 & \mathbf{E}_R \end{bmatrix}. \hspace{1cm} (20)$$

This matrix originates from $\mathbf{T}$ by the change of the modal submatrix $\mathbf{W}^{(F)}_D$ of the disk and the block diagonal matrix $\mathbf{W}_R$ for the unit matrices $\mathbf{E}_D$ of order $n_D^{(F)}$ and $\mathbf{E}_R$ of order $n_R$. 

4. APPLICATION

Presented method is tested on the experimental bladed disk developed in Institute of Thermomechanics, Academy of Science of the Czech Republic. As an illustration the Table 1 introduces six lowest natural frequencies of thereinbefore components. The number of the nodal diameters $d$ characterizes natural modes of the disk (number of nodal circuits of all presented modes is zero) and symbols $B_{xy}$, $B_{xz}$ (bending in marked plane) and $T$ (torsional) characterize the shape of blade natural modes. Results of following cases are introduced: (i) central clamped disk with flexible blade foots, (ii) central clamped disk with rigid blade foots, (iii) clamped blade with flexible shroud, (iv) clamped blade with rigid shroud.

<table>
<thead>
<tr>
<th>natural modes</th>
<th>natural frequencies of components in Hz</th>
<th>modes</th>
<th>blade (iii)</th>
<th>blade (iv)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>226.2, 234.8</td>
<td>$B_{xy}$ (1.)</td>
<td>135.9</td>
<td>135.6</td>
</tr>
<tr>
<td>2</td>
<td>226.2, 234.8</td>
<td>$B_{xz}$ (1.)</td>
<td>270.4</td>
<td>270.6</td>
</tr>
<tr>
<td>3</td>
<td>242.2, 249.8</td>
<td>$B_{xy}$ (2.)</td>
<td>928.5</td>
<td>958</td>
</tr>
<tr>
<td>4</td>
<td>266.3, 306.7</td>
<td>$T$ (1.)</td>
<td>1512</td>
<td>1523</td>
</tr>
<tr>
<td>5</td>
<td>266.3, 306.7</td>
<td>$B_{xz}$ (2.)</td>
<td>1825</td>
<td>1876</td>
</tr>
<tr>
<td>6</td>
<td>482.9, 599.3</td>
<td>$B_{xy}$ (3.)</td>
<td>2607</td>
<td>2786</td>
</tr>
</tbody>
</table>

5. CONCLUSION

The paper deals with the modelling of rotating flexible disk vibration with ideally fixed blades. The disk is modelled as three dimensional continuum discretized using isoparametric hexahedral solid finite elements and blades are modelled as one dimensional continuum with
the rigid shroud mounted at the end of blades. The method allows to take into account continuously distributed centrifugal and gyroscopic effects. The presented approach is based on the modal synthesis method and DOF number reduction corresponding to elastic displacements of the free disk nodes and blades. The displacements of the coupled disk nodes on rigid blade foots are eliminated by means of the first blade node displacements at blade roots.

The condensed model of the rotating bladed disk with relatively small DOF number is suitable for the simulations of vibrations influenced by slip contact interactions between shroud of particular blades. This new approach to the bladed disk vibration modelling will be tested on the physical model of a bladed disk developed in Institute of Thermomechanics, Academy of Science of the Czech Republic.

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REFERENCES


MODELOVÁNÍ KMITÁNÍ ROTUJÍCÍHO DISKU S VETKNUTÝMI LOPATKAMI

Summary. Příspěvek uvádí metodu matematického modelování kmitání rotujících olopatkovaných disků. Lopatky jsou uvažovány jako jednodimenzionální kontinua vetknutá do třídime ncionálního poddajného disku středově upevněného k rotoru turbíny, který rotuje konstantní úhlovou rychlostí. Dynamická analýza a optimalizace olopatkovaných disků s tlumícími elementy vyžaduje vyvinout efektivní metodu směřující k modelu olopatkovaného disku s relativně malým počtem stupňů volnosti. Takový model v budoucnu umožní simulovat nelineární kmitání celého systému ovlivněné interakcí kluzných kontaktních ploch ve vnitřních vazbách mezi lopatkami.